



Competency Based Questions

An organization conducted bike race under two different categories - Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions:



- (a) How many relations are possible from set B to set G ? Provide a proof.
- (b) Among the possible relations from B to G , how many functions can be formed from B to G ? Is this number the same as the number obtained in part (a)? If not, explain why?
- (c) Let $R: B \rightarrow B$ be defined by
- $$R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}.$$
- Check whether R is an equivalence relation.
- (d) A function $f: B \rightarrow G$ be defined by
- $$f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}.$$
- Determine whether f is bijective. If it is not bijective, what modification(s) are required to make it bijective?
- (e) A function $f: B \rightarrow G$ is chosen at random from all possible functions from B to G .
- (i) How many such functions are surjective?
- (ii) What is the probability that a randomly chosen function from $B \rightarrow G$ is onto?
- (f) Define a relation R on $B \cup G$ as follows:
- $$xRy \iff \text{either both } x, y \in B \text{ or both } x, y \in G.$$
- (i) Prove that R is an equivalence relation on $B \cup G$.
- (ii) Determine the equivalence classes.