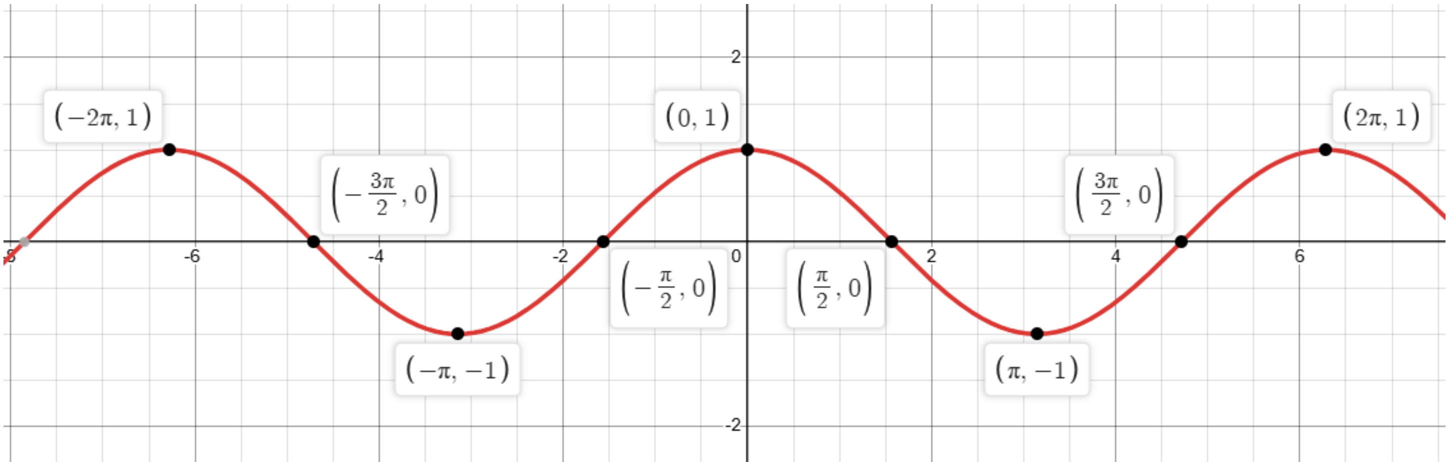
Competency Based Questions

Consider the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \cos x$.
A graph of the function is provided here for your reference.



(a) Is the function injective on $\mathbb{R}$? Why or why not.
If your answer is no, what can be done to make it injective?

(b) Is the function surjective on $\mathbb{R}$? Why or why not.
If your answer is no, what can be done to make it surjective?

(c) Is the function bijective on $\mathbb{R}$? Why or why not.
If your answer is no, what can be done to make it bijective?

(d) *Can you verify the results of (a), (b) and (c) analytically and with the help of the graph?*

(e) Consider the domain of $f(x) = \cos x$. Can you find subsets of this domain containing
(i) all angles for which the cosine value is 0,
(ii) all angles for which the cosine value is 1,
(iii) all angles for which the cosine value is −1, and
(iv) the remaining angles?

(f) Find all functions $h: \mathbb{R} \longrightarrow \mathbb{R}$ such that
$$h(x + y) = h(x) \cos y - \sin x \, h(y)$$
for all real x, y and $h(0) = 0$.

(g) Consider the domain of $h(x) = \sin x$. Can you find subsets of this domain containing
(i) all angles for which the cosine value is 0,
(ii) all angles for which the cosine value is 1,
(iii) all angles for which the cosine value is −1, and
(iv) the remaining angles?

(h) Find the number of solutions of the following equations
(i) $f(x) = x$
(ii) $f(x) = e^x$
(iii) $f(x) = x + \dfrac{1}{x}$

(i) A function f is said to be *even* if $f(-x) = f(x)$ and *odd* if $f(-x) = -f(x)$.
Express f and h as a sum of even and odd functions.