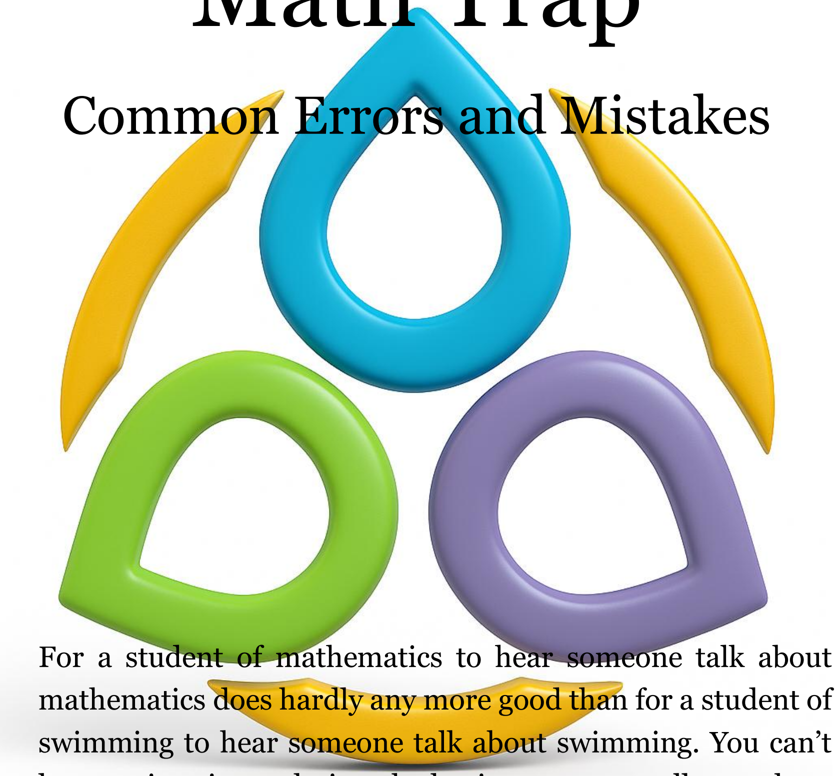
The

Math Trap

Common Errors and Mistakes



For a student of mathematics to hear someone talk about mathematics does hardly any more good than for a student of swimming to hear someone talk about swimming. You can't learn swimming technique by having someone tell you where to put your arms and legs; and you can't learn to solve problems by having someone tell you to complete the square or to substitute sin u for y.

P. R. Halmos

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The

Math Trap

Common Errors and Mistakes

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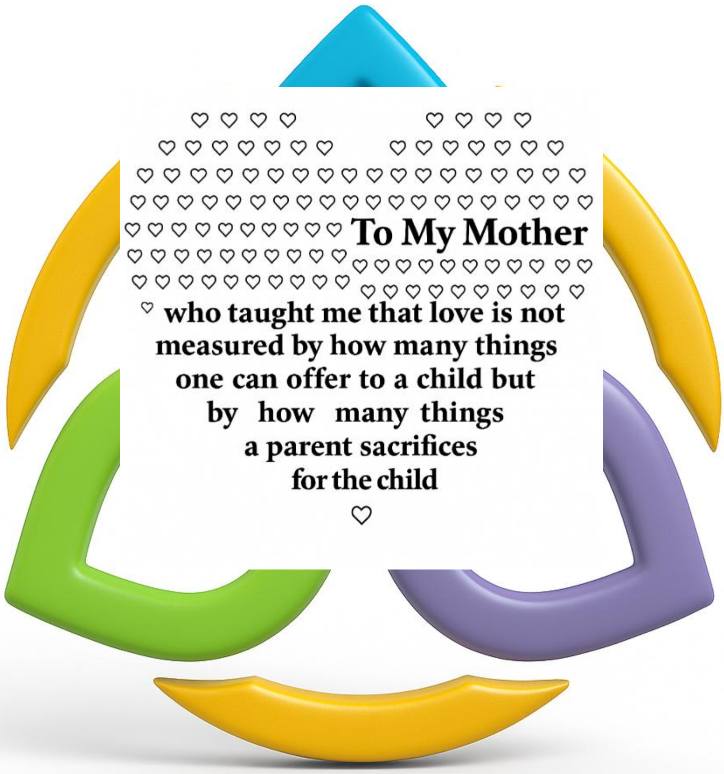
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who taught me that love is not
measured by how many things
one can offer to a child but
by how many things
a parent sacrifices
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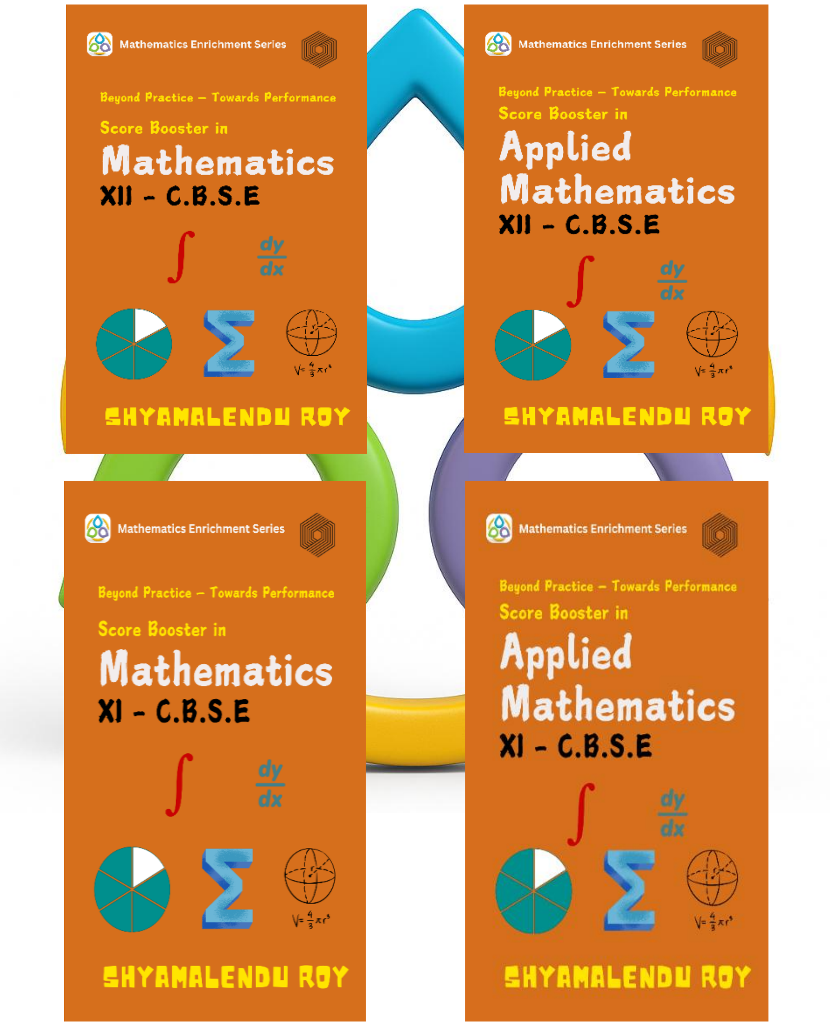

A Fresh Start . . .

Mathematics is often seen as a subject of precision, certainty, and exactness. Yet, anyone who has truly engaged with it knows that the path to understanding is rarely free from errors. In fact, mistakes are not obstacles—they are milestones. They show how we think, where we hesitate, and what we misunderstand. This book, *The Math Trap – Common Errors and Mistakes*, is built upon that very idea: that errors are not failures, but powerful tools for learning.

Over the years, as a teacher and learner of mathematics, I have observed that students across all levels tend to fall into similar *traps*. These traps may arise from misapplied formulas, overlooked conditions, careless calculations, or incomplete conceptual understanding. What makes them particularly interesting is that they often appear correct at first glance, making them even more deceptive. Identifying and understanding these traps is essential to developing clarity and confidence in mathematics.

This book is an attempt to bring such common errors into focus. It spans topics from arithmetic to calculus, highlighting mistakes that frequently occur in classrooms, examinations, and even advanced studies. Each example has been carefully chosen not merely to point out what went wrong, but to explain *why* it went wrong and *how* such errors can be avoided in the future. In doing so, the book encourages readers to think critically, question assumptions, and apply concepts with precision.

The purpose of this book is not to discourage mistakes, but to redefine them. A wrong step, when examined thoughtfully, can lead to a deeper and more lasting understanding than a correct answer achieved mechanically. By studying these errors, learners can sharpen their reasoning, strengthen their foundations, and cultivate a habit of careful thinking.

This work belongs as much to its readers as to its author. Despite careful preparation, a few errors may have escaped notice. Should you find any inaccuracies, I would greatly appreciate your feedback so that they may be corrected in future editions. You may reach me at

`math1089.9801@gmail.com`.

A heartfelt note of gratitude is owed to my daughters *Avirupa* and *Adrija* for the sacrifices they made during times I was unavailable to play or spend time with them. I also extend my sincere thanks to my family for their enduring patience and support throughout this journey.

May this book help you navigate the subtle traps of mathematics with awareness and insight, and may it inspire you to see every mistake as a step forward in your journey of learning.

Happy exploring!

Shyamalendu Roy
Akshaya Tritiya
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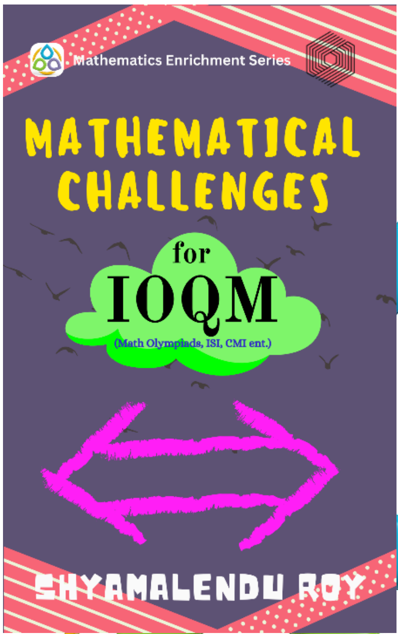

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1 Where Fractions go Wrong

> A man is like a fraction, whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction.
>
> Leo Tolstoy

Fractions are among the first abstract concepts students encounter in mathematics. They look simple, just a numerator over a denominator, but they often hide traps that catch even the sharpest minds.

In this chapter, we explore the most common pitfalls students and even seasoned problem-solvers fall into when dealing with fractions. From converting a fraction to misapplying cross-multiplication, from flawed simplifications to algebraic missteps—we'll break down where things go wrong, why they go wrong, and most importantly, how to avoid these traps.

1.1. Importance of Equal Parts

One of the most fundamental—but often misunderstood—ideas in fractions is the concept of *equal parts*. The denominator in a fraction doesn't just tell us how many parts a whole is divided into; it tells us that *these parts must be equal*. Yet, many students focus only on the number of parts and ignore the crucial condition of *equality*.

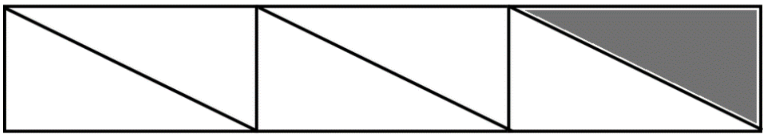
The impact becomes clear when students are asked to draw or interpret fractions visually. Without a solid grasp of equal partitioning, their representations often miss the mark—showing parts that are uneven or arbitrarily divided. In this section, we'll highlight why understanding equal parts is the key to mastering fractions.

(**a**) This is not $\frac{1}{2}$ (**b**) This is not $\frac{1}{4}$ (**c**) This is not $\frac{3}{4}$

In the *first* figure, the figure is not divided into two equal parts. Therefore, the shaded part is not 1/2. In the *second* case, the figure is not divided into four equal parts. Therefore, the shaded part is not 1/4. Similarly, the figure is not divided into equal four parts in the *third* figure. So, the shaded part is different from 3/4. The diagram below presents the correct representation.

(**a**) This is $\frac{1}{2}$ (**b**) This is $\frac{1}{4}$ (**c**) This is $\frac{3}{4}$

When asked to identify the fraction representing the shaded area, many students misunderstood the concept of a fraction as part of a whole. Some said 1/2, viewing the shaded part as half of the last rectangle. Others said 1/3, focusing on three triangles without considering their internal divisions. A few choose 1/5, counting one shaded and five unshaded parts. However, the correct answer is 1/6, as the whole is divided into six equal parts, and only one part is shaded.



1.2. Importance of Common Denominator

Here are some examples that highlight why a common denominator is important when calculating with fractions. This is especially important when adding or subtracting two fractions.

[1] Addition performed without using a *common denominator*:

$$\frac{3}{4}+\frac{1}{3}=\frac{3+1}{4+3}=\frac{4}{7}$$

Clearly, the result is not true. The student added the numerators and then the denominators separately treating the numerators and denominators as independent whole numbers, without finding the common denominator.

The correct method should go as follows:

$$\begin{aligned}&\frac{3}{4}+\frac{1}{3}\\&=\frac{3\times 3}{4\times 3}+\frac{1\times 4}{3\times 4}\\&=\frac{9}{12}+\frac{4}{12}\\&=\frac{9+4}{12}\\&=\frac{13}{12}\\&=1\frac{1}{12}\end{aligned}$$

OR

$$\frac{3}{4}+\frac{1}{3}$$
$$=\frac{(12\div 4)\times 3+(12\div 3)\times 1}{12}$$
$$=\frac{3\times 3+4\times 1}{12}$$
$$=\frac{9+4}{12}$$
$$=\frac{13}{12}$$
$$=1\frac{1}{12}$$

OR

Butterfly method

$$\frac{3}{4}+\frac{1}{3}$$
$$=\frac{3\times 3+4\times 1}{4\times 3}$$
$$=\frac{9+4}{12}$$
$$=\frac{13}{12}$$
$$=1\frac{1}{12}$$

[2] Subtracting fractions without finding a common denominator

$$\frac{5}{8}-\frac{3}{7}=\frac{5-3}{8-7}=\frac{2}{1}=2$$

Clearly, the result is not true. The user subtracted the numerators and the denominators separately, treating the numerators and denominators as independent whole numbers, without finding the common denominator.

The correct method should go as follows:

$$\frac{5}{8}-\frac{3}{7}$$
$$=\frac{5\times 7}{8\times 7}-\frac{3\times 8}{3\times 8}$$
$$=\frac{35}{56}-\frac{24}{56}$$
$$=\frac{35-24}{56}$$
$$=\frac{11}{56}$$

OR

$$\frac{5}{8}-\frac{3}{7}$$
$$=\frac{(56\div 8)\times 5}{56}-\frac{(56\div 7)\times 3}{56}$$
$$=\frac{7\times 5}{56}-\frac{8\times 3}{56}$$
$$=\frac{35}{56}-\frac{24}{56}$$
$$=\frac{35-24}{56}$$
$$=\frac{11}{56}$$

OR

Butterfly method

$$\frac{5}{8}-\frac{3}{7}$$
$$=\frac{5\times 7-3\times 8}{8\times 7}$$
$$=\frac{35-24}{56}$$
$$=\frac{11}{56}$$

1.3. Common Mistakes in Multiplication and Division

This section is devoted to a few common mistakes that occur during the multiplication and division of fractions. This is not an exhaustive list—you may add more examples whenever you find them. Also, simplify the fractions wherever possible.

[3] Incorrect multiplication caused by cross-multiplication confusion

$$\frac{2}{5} \times \frac{4}{6} = \frac{2 \times 6}{5 \times 4} = \frac{12}{20}$$

The student multiplies the numerator of one fraction by the denominator of the other, treating it like a cross-multiplication problem. This shows a misunderstanding of the correct rule for multiplying fractions. The correct method is as follows:

$$\frac{2}{5} \times \frac{4}{6} = \frac{2 \times 4}{5 \times 6} = \frac{8}{30}$$

[4] Incorrect cancellation of fractions

$$\frac{\cancel{2}}{5} \times \frac{\cancel{4}}{6}$$

$$= \frac{1 \times 2}{5 \times 6}$$

$$= \frac{2}{24}$$

Clearly, this is false. The correct method is as follows:

$$\frac{\cancel{2}}{5} \times \frac{4}{\cancel{6}}$$

$$= \frac{1 \times 4}{5 \times 3}$$

$$= \frac{4}{15}$$

[5] Skipped finding the reciprocal before division

$$\frac{5}{6} \div \frac{7}{8} = \frac{5}{6} \times \frac{7}{8} = \frac{5 \times 7}{6 \times 8} = \frac{35}{48}$$

This is clearly false. While dividing fractions, we should keep three things in mind:

- **a.** The first fraction remains unchanged.
- **b.** The division sign (÷) changes to a multiplication sign (×)
- **c.** The second fraction is replaced by its reciprocal.

In view of these points, the correct method is as follows:

$$\frac{5}{6} \div \frac{7}{8}$$
$$= \frac{5}{6} \times \frac{8}{7}$$
$$= \frac{5 \times 8}{6 \times 7}$$
$$= \frac{40}{42}$$

[6] A few common mistakes related to the above question are shown below.

$$\frac{5}{6} \div \frac{7}{8} = \frac{6}{5} \times \frac{7}{8}$$
$$= \frac{6 \times 7}{5 \times 8} = \frac{42}{40}$$

OR

$$\frac{5}{6} \div \frac{7}{8} = \frac{6}{5} \times \frac{8}{7}$$
$$= \frac{6 \times 8}{5 \times 7} = \frac{48}{35}$$

Solve them using the correct method.
Further simplification is definitely possible in the above cases, wherever applicable.

[7] The following example illustrates a student's misconception, where the addition rule was incorrectly applied while solving a multiplication problem involving fractions. The student multiplied only the numerators but not the denominators—another case of applying the wrong rule.

$$\frac{6}{9} \times \frac{7}{9} = \frac{6 \times 7}{9} = \frac{42}{9}$$

1.4. More Common Mistakes

In this section, we will look at a few examples of common mistakes. These are mixed in nature and include addition, subtraction, multiplication, and division. You may add more examples to this list as you come across them.

[8] Add the numerators treating them as independent whole numbers and picked the denominator.

$$\frac{2}{10} + \frac{1}{2} = \frac{2+1}{10} = \frac{3}{10}$$

OR

$$\frac{3}{7} + \frac{5}{16} = \frac{3+5}{16} = \frac{8}{16}$$

[9] Subtract the numerators treating them as independent whole numbers and picked the denominator.

$$\frac{5}{12} - \frac{3}{7} = \frac{5-3}{12} = \frac{2}{12}$$

OR

$$\frac{5}{3} - \frac{4}{12} = \frac{5-4}{12} = \frac{1}{12}$$

[10] Add the numerators treating them as independent whole numbers but computed the denominator using multiplication operation.

$$\frac{3}{10}+\frac{1}{2}=\frac{3+1}{10\times 2}=\frac{4}{20}$$

[11] Add the numerators treating them as independent whole numbers but computed the denominator using division operation.

$$\frac{3}{10}+\frac{1}{2}=\frac{3+1}{10\div 2}=\frac{4}{5}$$

[12] Subtract the numerators treating them as independent whole numbers but computed the denominator using multiplication operation.

$$\frac{3}{10}-\frac{1}{2}=\frac{3-1}{10\times 2}=\frac{2}{20}$$

[13] Subtract the numerators treating them as independent whole numbers but computed the denominator using division operation.

$$\frac{3}{10}-\frac{1}{2}=\frac{3-1}{10\div 2}=\frac{2}{5}$$

[14] Combines denominators and numerators into a whole number by adding all the numbers together or observes no distinction between numerator and denominator. The resulting answer is a whole number.

$$\frac{2}{10}+\frac{1}{2}=312$$

[15] Add the numerator and denominator in the first fraction to become the numerator in the answer and add the numerator and denominator of the second fraction to become the denominator in the answer.

$$\frac{5}{12}+\frac{3}{7}=\frac{5+12}{12+7}=\frac{17}{19}$$

[16] Consider the following equality:

$$\frac{2}{2} = \frac{3}{3}$$

Taking out the commons 2 and 3 from the LHS and RHS, we get

$$2\left(\frac{1}{1}\right) = 3\left(\frac{1}{1}\right)$$

Cancelling the factor (1/1) from both sides, we get 2 = 3. Clearly, this is false. Where is the mistake?

[17] What is the meaning of $2\frac{3}{7}$? Is it same as $2 \times \frac{3}{7}$?

Of course not. Here 2 represents the whole part and $\frac{3}{7}$ represents the fractional part in this number.

The correct way to calculate this is

$$2\frac{3}{7} = \frac{7 \times 2 + 3}{7} = \frac{17}{7}.$$

Of course,

$$2 \times \frac{3}{7} = \frac{2 \times 3}{7} = \frac{6}{7}$$

and they are not equal.

1.6. A Few Coincidences

Below, we will present a few examples in which the correct answer was obtained despite some incorrect intermediate steps.

[20] Let's work out the following subtraction of fractions:

$$\frac{9}{2} - \frac{25}{10}$$
$$= \frac{9 \times 5}{2 \times 5} - \frac{25}{10}$$

$$= \frac{45}{10} - \frac{25}{10}$$
$$= \frac{45 - 25}{10}$$
$$= \frac{20}{10} = 2$$

If we solve it in the following way, we get

$$\frac{9}{2} - \frac{25}{10} = \frac{9 - 25}{2 - 10} = \frac{-16}{-8} = 2$$

Surprisingly, we get the correct answer. This is mere coincidence! Can you produce one more example?

[22] Consider the following expressions:

$$\sqrt{3\frac{5}{6}} = \sqrt{\frac{6 \times 3 + 5}{6}}$$
$$= \sqrt{\frac{23}{6}}$$

and

$$3\sqrt{\frac{5}{6}} = \sqrt{9 \times \frac{5}{6}}$$
$$= \sqrt{\frac{9 \times 5}{6}} = \sqrt{\frac{45}{6}}$$

Of course,

$$\sqrt{3\frac{5}{6}} \neq 3\sqrt{\frac{5}{6}}$$

On the other hand,

$$\sqrt{2\frac{2}{3}} = \sqrt{\frac{2 \times 3 + 2}{3}} = \sqrt{\frac{8}{3}}$$

and

$$2\sqrt{\frac{2}{3}} = \sqrt{4 \times \frac{2}{3}}$$

$$= \sqrt{\frac{4 \times 2}{3}} = \sqrt{\frac{8}{3}}$$

Of course,

$$\sqrt{2\frac{2}{3}} = 2\sqrt{\frac{2}{3}}$$

[25] Here are a few more coincidences. Can you produce a proof?

$$\frac{2^2 + 86^2}{15^2 + 95^2} = \frac{2 + 86}{15 + 95}$$

$$\frac{7^3 + 41^3}{29^3 + 43^3} = \frac{7 + 41}{29 + 43}$$

$$\frac{3^4 + 6^4 + 18^4}{5^4 + 16^4 + 17^4} = \frac{3 + 6 + 18}{5 + 16 + 17}$$

1.7. An Unusual Multiplication

Two fractions can be multiplied theoretically as follows:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

For example,

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}.$$

Consider an instance where a student multiplied two fractions as

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

and produced an example like

$$\frac{4}{1}\times\frac{5}{8}=\frac{45}{18}.$$

In this example, we have 45 on the numerator and 18 on the denominator. And the result is obtained by just concatenating the given numbers (in that order) in the left side.
Actual multiplication suggests that LHS is equal to

$$\frac{4}{1}\times\frac{5}{8}=\frac{4\times 5}{1\times 8}=\frac{20}{8}=\frac{5}{2}$$

after simplification. And RHS is equal to

$$\frac{45}{18}=\frac{5}{2}$$

so that the equality holds. This way of fraction multiplication is merely a coincidence. If we simply interchange the fractions and looking for their equality, it won't hold!

For example,

$$\frac{5}{8}\times\frac{4}{1}=\frac{5\times 4}{8\times 1}=\frac{20}{8}=\frac{5}{2}$$

and

$$\frac{54}{81}=\frac{2}{3}$$

certainly tells us that

$$\frac{5}{8}\times\frac{4}{1}\neq\frac{54}{81}$$

A few more examples of such irrational multiplications are given below:

$$\frac{1}{2}\times\frac{5}{4}=\frac{15}{24} \qquad \frac{1}{6}\times\frac{4}{3}=\frac{14}{63}$$

$$\frac{4}{9}\times\frac{9}{8}=\frac{49}{98} \qquad \frac{9}{1}\times\frac{5}{9}=\frac{95}{19}$$

Can you find more such instances?

1.8. Howlers

Consider the fraction $\frac{26}{65}$. Simplified value of this fraction is $\frac{2}{5}$. This can be obtained in the following ways:

■ Divide both numerator and denominator by the common factor 13, which is ok.

$$\frac{26}{65} = \frac{26 \div 13}{65 \div 13} = \frac{2}{5}.$$

■ Cancel the digit 6 present both in numerator and denominator and get the correct result.

$$\frac{26}{65} = \frac{2\not{6}}{\not{6}5} = \frac{2}{5}.$$

Of course, this is not true in general. In fact, not all the fractions follow this rule. For example,

$$\frac{14}{47} \neq \frac{1}{7} \text{ and } \frac{45}{57} \neq \frac{4}{7}.$$

A few examples of such fractions (consisting of two digits) which can be reduced similarly are

$$\frac{16}{64}, \frac{26}{65}, \frac{19}{95}, \text{ and } \frac{49}{98}.$$

You may now wonder, if there are fractions composed of more than two digits, where this strange type of cancellation holds true. Here are a few examples with the fractions of three digits.

$$\frac{138}{345} = \frac{18}{45}$$
$$\frac{163}{326} = \frac{1}{2}$$
$$\frac{332}{830} = \frac{32}{80}$$

$$\frac{499}{998} = \frac{4}{8}$$
$$\frac{166}{664} = \frac{1}{4}$$

Consider the fraction $\frac{26}{65}$ once again. In view of the examples given above, we can have the following surprising results as well.

$$\frac{26}{65} = \frac{266}{665} = \frac{2666}{6665} = \frac{26666}{66665} = \frac{2}{5}$$

A pattern is beginning to take shape, and you might notice that the following examples also fit within this framework:

$$\frac{16}{64} = \frac{166}{664} = \frac{1666}{6664} = \frac{16666}{66664} = \frac{1}{4}$$
$$\frac{19}{95} = \frac{199}{995} = \frac{1999}{9995} = \frac{19999}{99995} = \frac{1}{5}$$
$$\frac{49}{98} = \frac{499}{998} = \frac{4999}{9998} = \frac{49999}{99998} = \frac{4}{8}$$

A few more examples are given below of large fractions.

$$\frac{484848484}{848484847} = \frac{4}{7} \qquad \frac{6486486}{8648648} = \frac{6}{8}$$

1.10. Printer's Errors

Printer's errors sometimes occur when exponents or multiplication signs are omitted, yet the resulting expression remains equivalent to the intended one. For example:

$$2^5 \cdot \frac{25}{31} = 25\frac{25}{31}$$

Certainly, each side is equal to

$$\frac{800}{31}$$

2 Exposing Errors in Surds and Indices

> There is no science which teaches the harmonies of nature more clearly than mathematics.
>
> Paul Carus

Exponents are not just shortcuts for repeated multiplication; they show important patterns. Even a small mistake in their rules can lead to big errors. Surds may look difficult, but most mistakes happen when we treat them like ordinary numbers, especially while adding or simplifying them.

[10] We all know that 0^0 is indeterminate.

In the following example, we will show that $0^0 = 1$. We have

$$0^0 = (a-a)^{n-n}$$

$$\Rightarrow 0^0 = \frac{(a-a)^n}{(a-a)^n}$$

$$\Rightarrow 0^0 = 1$$

Can you find out the error in this calculation?

2.2. A few Lucky Mistakes

[11] Consider the following expressions:

$$\left(\frac{8}{2}\right)^{\frac{1}{2}} = (4)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = (2)^{2\cdot\frac{1}{2}} = 2^1 = 2$$

$$\frac{8^1}{2^2} = \frac{8}{4} = 2$$

It follows that

$$\left(\frac{8}{2}\right)^{\frac{1}{2}} = \frac{8^1}{2^2},$$

which is clearly a mere coincidence.
In general, we can restate this as

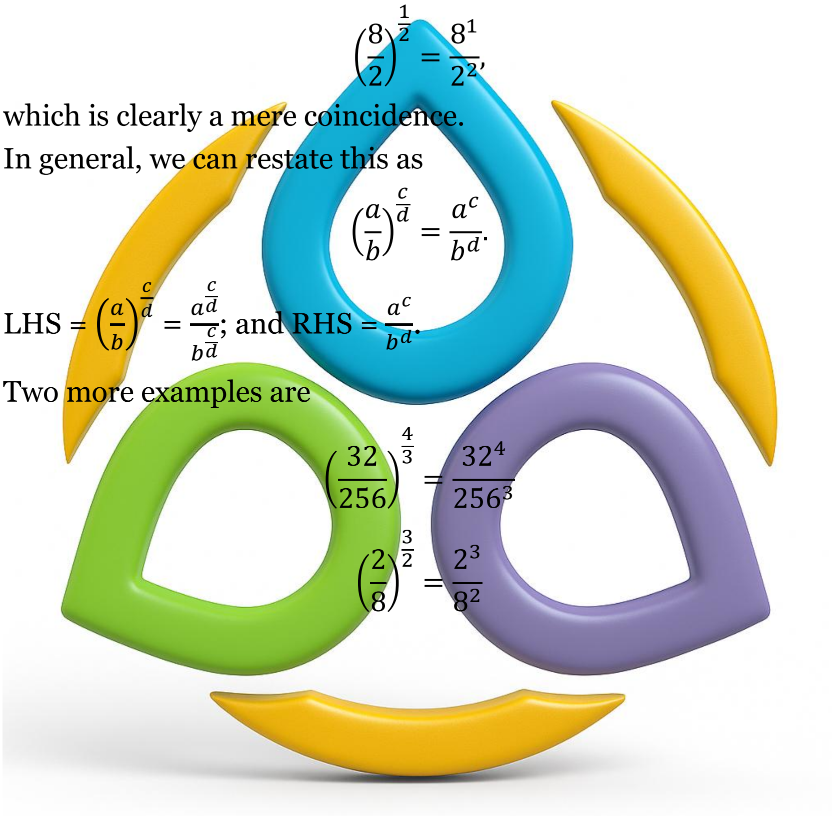
$$\left(\frac{a}{b}\right)^{\frac{c}{d}} = \frac{a^c}{b^d}.$$

LHS = $\left(\frac{a}{b}\right)^{\frac{c}{d}} = \frac{a^{\frac{c}{d}}}{b^{\frac{c}{d}}}$; and RHS = $\frac{a^c}{b^d}$.

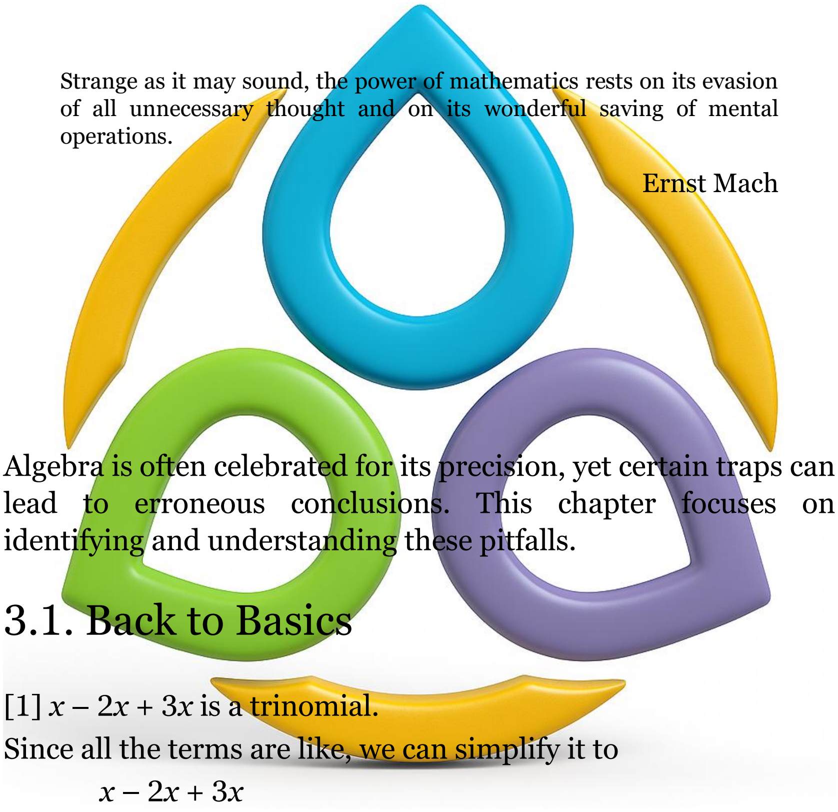
Two more examples are

$$\left(\frac{32}{256}\right)^{\frac{4}{3}} = \frac{32^4}{256^3}$$

$$\left(\frac{2}{8}\right)^{\frac{3}{2}} = \frac{2^3}{8^2}$$



3 The Traps of Algebra



> Strange as it may sound, the power of mathematics rests on its evasion of all unnecessary thought and on its wonderful saving of mental operations.
>
> Ernst Mach

Algebra is often celebrated for its precision, yet certain traps can lead to erroneous conclusions. This chapter focuses on identifying and understanding these pitfalls.

3.1. Back to Basics

[1] $x - 2x + 3x$ is a trinomial.

Since all the terms are like, we can simplify it to

$$x - 2x + 3x$$
$$= 4x - 2x$$
$$= 2x,$$

which is a monomial.

In fact, a monomial contains three unlike terms.

3.2. Wrong Application of Distributive Laws

Distributive laws are extremely important concepts in algebra. It has immense applications in other fields as well. In this section we will explore various misconceptions in applying distributive property. Recall the distributive laws:

$$a \times (b + c) = a \times b + a \times c$$
$$(b + c) \times a = b \times a + c \times a$$

and

$$a \times (b - c) = a \times b - a \times c$$
$$(b - c) \times a = b \times a - c \times a$$

[4] Our first example from the distributive law:

$$2(x + 7) = 2x + 7$$

This is clearly false. Actual answer is

$$2(x + 7) = 2x + 2 \times 7$$
$$= 2x + 14.$$

[5] Another example of common error is

$$3(x - 4) - 4(x - 1)$$
$$= 3x - 4 - 4x - 1$$

Other variations of this particular example could be

$$3(x - 4) - 4(x - 1)$$
$$= 3x - 3 \times 4 - 4x - 1 \times 4$$

and

$$3(x - 4) - 4(x - 1)$$
$$= 3x - 3 \times 4 - 4x + 1$$

While the actual answer must be

$$3(x - 4) - 4(x - 1)$$
$$= 3x - 3 \times 4 - 4x + 1 \times 4$$
$$= 3x - 12 - 4x + 4$$

3.5. Freshman's Dream

The freshman's dream (or exponentiation) is the generally-false equation $(x + y)^n = x^n + y^n$. Students commonly make this error in computing the power of a sum of real numbers. They falsely assume that powers distribute over sums. The correct result is given by the binomial theorem, which has additional terms in the middle when $n \geq 2$.

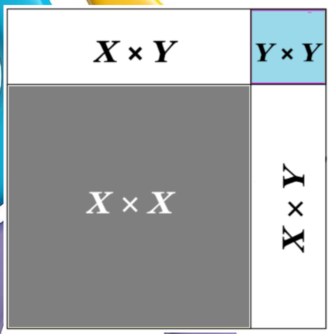
[12] A popular mistake in algebra is to write $(x + y)^2 = x^2 + y^2$.
If this need to be true, then we have

$$x^2 + y^2 + 2xy = x^2 + y^2$$
$$\text{or, } 2xy = 0$$

It follows that, either $x = 0$ or $y = 0$ or $x = 0 = y$.
Clearly, $(4 + 0)^2 = 4^2 + 0^2$ and $(0 + 5)^2 = 0^2 + 5^2$ are true.
Certainly, $(4 + 5)^2 = 4^2 + 5^2$ is false. Because

$$(4 + 5)^2 = 4^2 + 5^2$$
$$\Rightarrow (9)^2 = 4^2 + 5^2$$
$$\Rightarrow 81 = 16 + 25$$
$$\Rightarrow 81 = 41$$

is obviously absurd.



An illustration of the Freshman's dream in two dimensions. Each side of the square is $(X + Y)$ in length.
The area of the bigger square is the sum of the areas of two square regions ($= X^2$) and ($= Y^2$) and the area of the two white regions ($= 2 \times X \times Y$).

[13] A similar mistake in algebra is to write $(x + y)^3 = x^3 + y^3$.
If this need to be true, then we have

$$x^3 + y^3 + 3xy\,(x + y) = x^3 + y^3$$
$$\text{or, } 3xy\,(x + y) = 0$$

It follows that, either $x = 0$ or $y = 0$ or $x = -y$.
Similarly, as above $(4 + 0)^3 = 4^3 + 0^3$ and $(0 + 5)^3 = 0^3 + 5^3$ are true.
Moreover, $[4 + (-4)]^3 = 4^3 + (-4)^3$ is also true.
But, $(4 + 1)^3 = 4^3 + 1^3$ and $(2 + 5)^3 = 2^3 + 5^3$ are false.

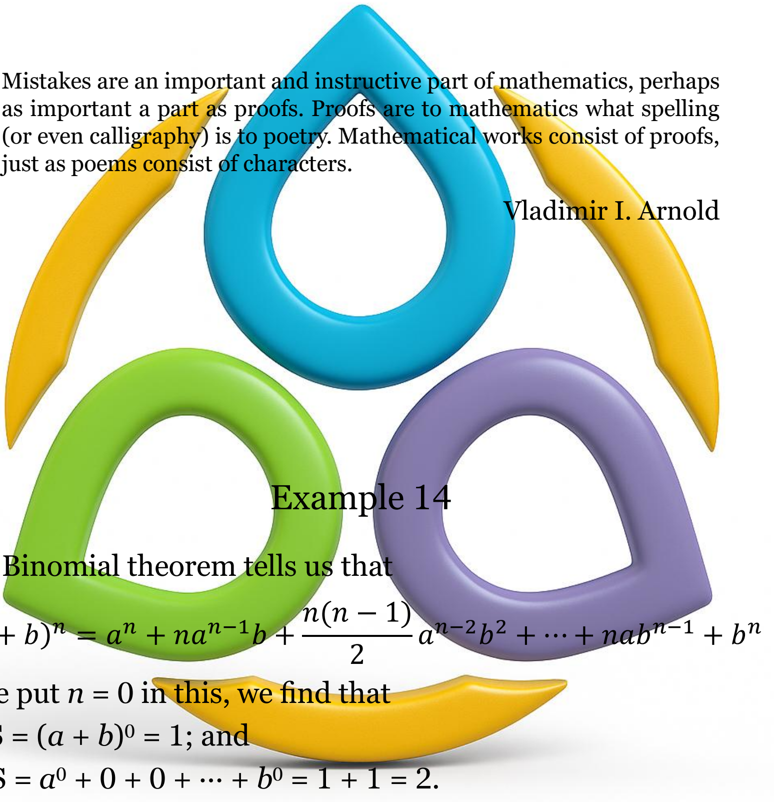
4 Common Mistakes in Trigonometry

We have overcome the notion that mathematical truths have an existence independent and apart from our own minds. It is even strange to us that such a notion could ever have existed.

Edward Kasner and James R. Newman

5

1 = 2
The Illusion of Logic



Mistakes are an important and instructive part of mathematics, perhaps as important a part as proofs. Proofs are to mathematics what spelling (or even calligraphy) is to poetry. Mathematical works consist of proofs, just as poems consist of characters.

Vladimir I. Arnold

Example 14

The Binomial theorem tells us that

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + nab^{n-1} + b^n$$

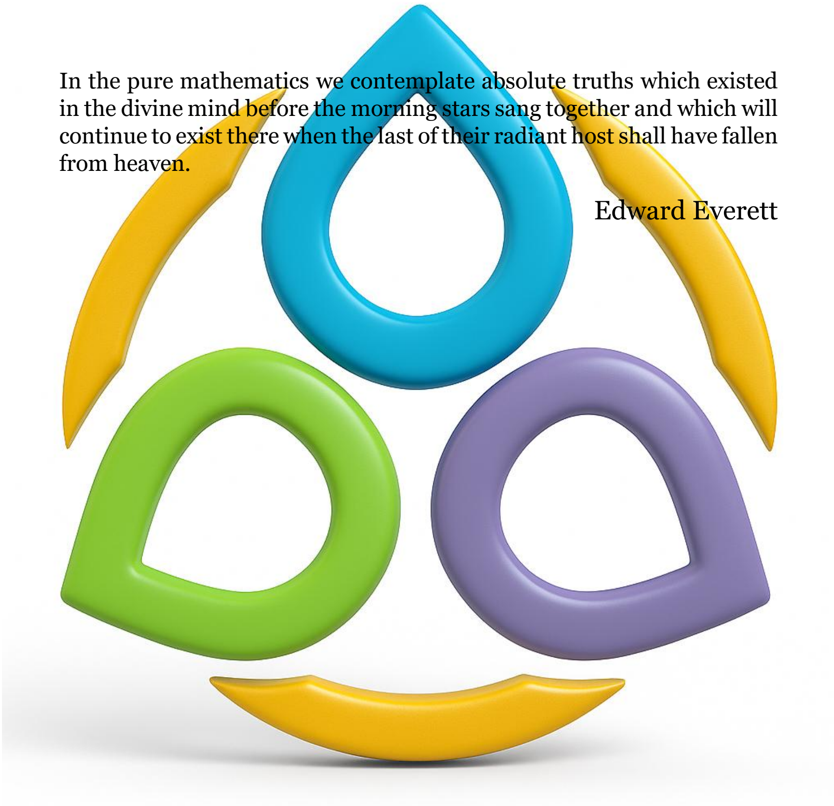
If we put $n = 0$ in this, we find that

LHS = $(a+b)^0 = 1$; and

RHS = $a^0 + 0 + 0 + \cdots + b^0 = 1 + 1 = 2$.

Therefore, we conclude that $1 = 2$.

6 Misconceptions in Complex Numbers



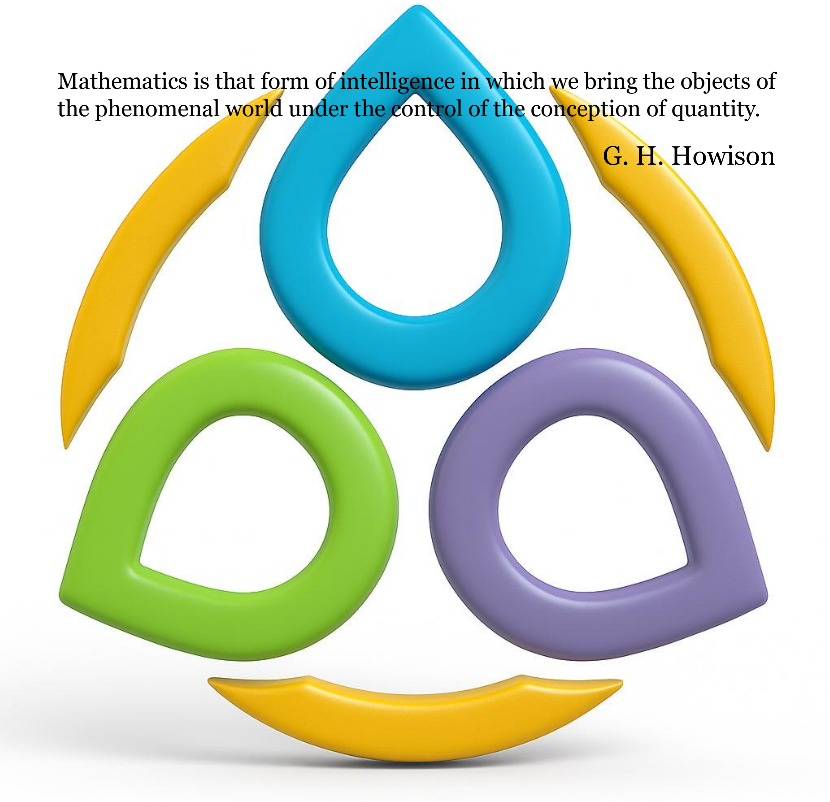
In the pure mathematics we contemplate absolute truths which existed in the divine mind before the morning stars sang together and which will continue to exist there when the last of their radiant host shall have fallen from heaven.

Edward Everett

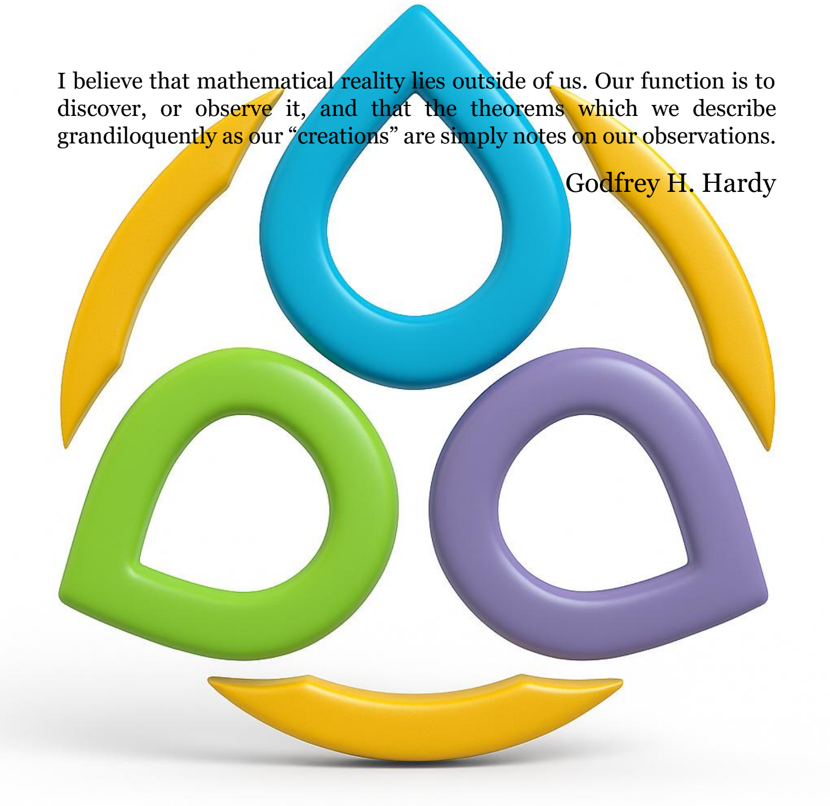
7 Graphs as Tools to Detect Errors

Mathematics is that form of intelligence in which we bring the objects of the phenomenal world under the control of the conception of quantity.

G. H. Howison



8 Pitfalls in Geometry



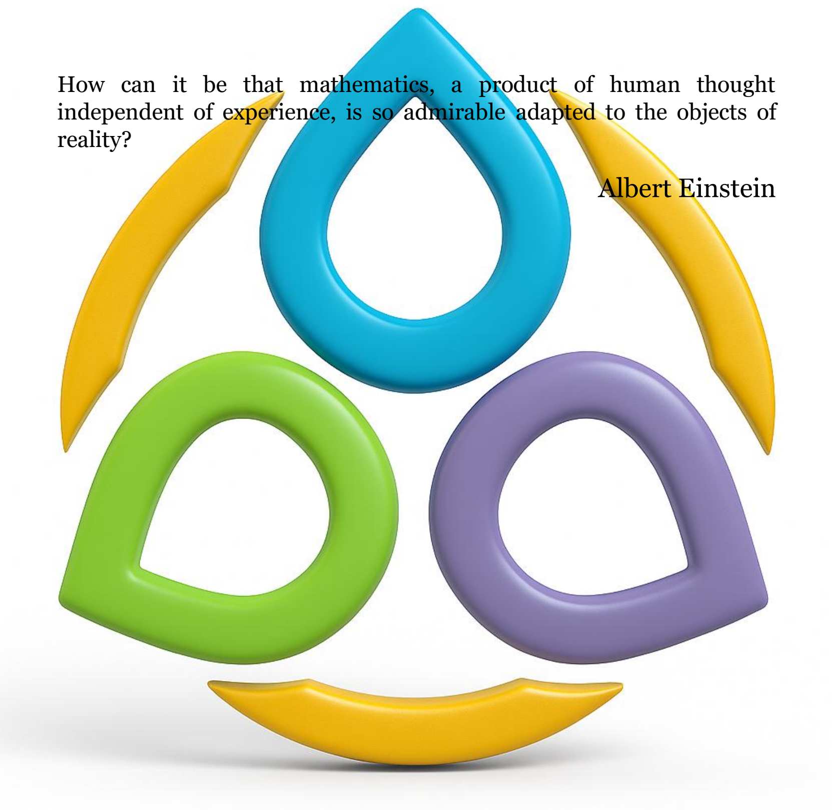
I believe that mathematical reality lies outside of us. Our function is to discover, or observe it, and that the theorems which we describe grandiloquently as our "creations" are simply notes on our observations.

Godfrey H. Hardy

9 When Calculus Misleads

How can it be that mathematics, a product of human thought independent of experience, is so admirable adapted to the objects of reality?

Albert Einstein



10 Mixed Bag

There is no branch of mathematics, however abstract, which may not someday be applied to phenomena of the real world.

Nikolai Lobachevsky